A SIMPLE WAY TO TEACH LOGARITHMS

We have taught logarithms in high school and college algebra and in introductory calculus courses. Before we began using the method of teaching logs described in this article, we found that many students had difficulties mastering the concept, more so than with other functions. Other teachers reported similar experiences.

The conceptual way to understand the function $y = \log_a x$ is to view it as the inverse of $y = a^x$. When students are beginning to learn functions, they can get lost in the details of working with inverses. Our approach is nothing more than a simple change of notation: we replace $\log_a$ by $a^\square$, read "*a*-box." We have used this device in our classes and have found a great improvement in the students' comprehension of logarithms.

Beginning with examples rather than general definitions, we write the following equation on the chalkboard:

We begin with some examples to establish a pattern

$$2^\square(8) = \_\_\_\_ ?$$

We say, "Two-box of 8 equals blank. What number goes in the box so that 2 raised to that power is 8?" Since $2^3 = 8$, we fill in the blank with a 3:

$$2^\square(8) = \underline{\ 3\ }.$$

We ask the class to supply the answers to the next few examples:

$$2^\square(16) = 4$$
$$2^\square(2) = 1$$
$$3^\square(27) = 3$$
$$3^\square(9) = 2$$

By the fifth example, everyone in the room is chiming in, "Three-box of 9 is 2."

Next, we introduce some examples that require a little more thought, for example,

$$2^\square\left(\frac{1}{2}\right) = \_\_\_\_ ?$$

After giving the class a few moments to consider the answer, we explain that because

$$2^{-1} = \frac{1}{2},$$

then

$$2^\square\left(\frac{1}{2}\right) = \underline{\ -1\ }.$$

We do some more examples with negative and fractional answers, carefully explaining how to use the appropriate properties of exponents.

$$2^\square(\sqrt{2}) = \left(\frac{1}{2}\right)$$
$$3^\square\left(\frac{1}{9}\right) = -2$$
$$8^\square(2) = \frac{1}{3}$$
$$2^\square\left(\frac{1}{16}\right) = -4$$

Everyone in the class usually understands the pattern after a few examples. At this time, we point out some general properties of the function $y = a^\square(x)$.

As we present the following examples one at a time, we ask students to reason them out before giving the answer.

$$2^\square(1) = 0$$
$$3^\square(1) = 0$$

The observation here is that no matter what a is, $a^\square(1) = 0$, since $a^0 = 1$. Then we ask for the answer to this equation:

$$2^\square(-2) = \_\_\_\_ ?$$

Since 2 raised to *any* exponent is positive—we like to reinforce this fact by pointing to the graph of $y = 2^x$—we see that $2^\square(-2)$ cannot exist. Indeed, for *any* a greater than 0, $a^\square(x)$ does not exist for x less than or equal to 0. Here it is appropriate to draw the graphs of examples of this new function, say, $y =$

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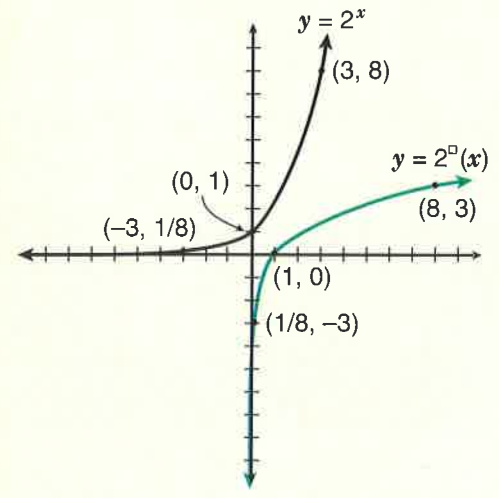

Fig. 1
The functions $y = 2^x$ and $y = 2^{\square}(x)$

$2^{\square}(x)$ and $y = 3^{\square}(x)$, pointing out their common features: the x-intercept is (1, 0), the y-axis is a vertical asymptote, and their domain excludes the negative numbers and 0 (see **fig. 1**). We also establish the essential connection with the exponential functions by demonstrating that $y = 2^{\square}(x)$ is the inverse of $y = 2^x$.

Finally, we announce that another way to write $2^{\square}(x)$ is $\log_2(x)$ and, in general, $a^{\square} = \log_a$. Again, a few examples are in order.

$$\log_3(81) = 3^{\square}(81) = 4$$

$$\log_4(2) = 4^{\square}(2) = \frac{1}{2}$$

$$\log_2(-1) = 2^{\square}(-1) \text{ (does not exist)}$$

Since "two-box" is not a scary function, neither is this strange new word, *logarithm*. In short, the a-box notation is a bridge to the standard $\log_a$ notation. After some practice, a-box notation can be dropped entirely.

The nice feature about a-box notation is that it makes transparent the evaluation of the log function. It can be used to establish the basic properties of logs. For example, $a^{\square}(a^x) = x$. Translating into standard notation, we have the familiar rule

$$\log_a (a^x) = x.$$

This presentation can be embellished and adapted to fit diverse learning and teaching styles. In fact, the box idea can be used for any inverse function. For example, the concept of arcsin can be explained by using sin □.

We have had tremendous success using the a-box technique to teach logarithms. It is our hope that others will try it and that their students will benefit from it. We taught a college-algebra course in which an identical quiz on logs was given to three sections of the same course. In one section, the a-box presentation had been used; in the other two sections, logarithms had been presented with only the standard notation. The students in the a-box section evaluated the logs on the quiz with 100 percent accuracy. Most of them translated from $\log_a$ to a-box to find the result. Students in the other two sections showed the usual wide range of confusions we commonly see with logs. We welcome reports of successes or criticisms from readers who try this method in their classes.

Using our notation helps establish the properties of logs

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